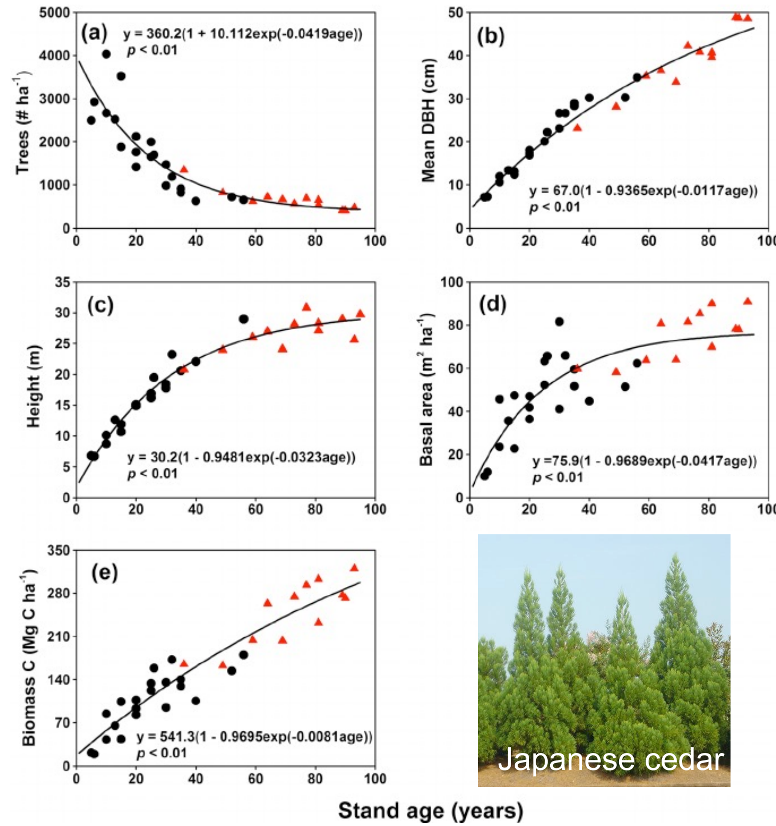
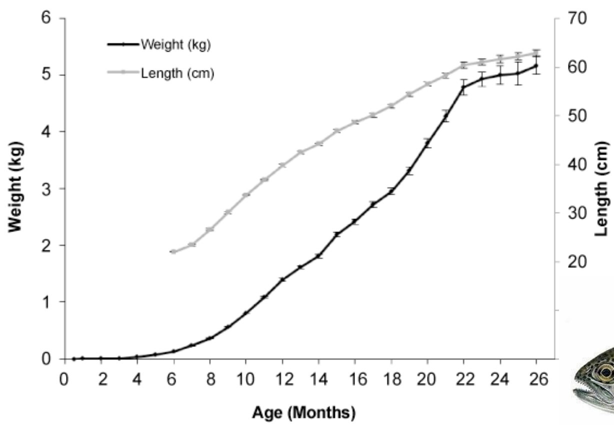
The importance of age structure

- biomass of individuals, be they animals or plants, increases with age
- so does their value
- we will neglect density dependence





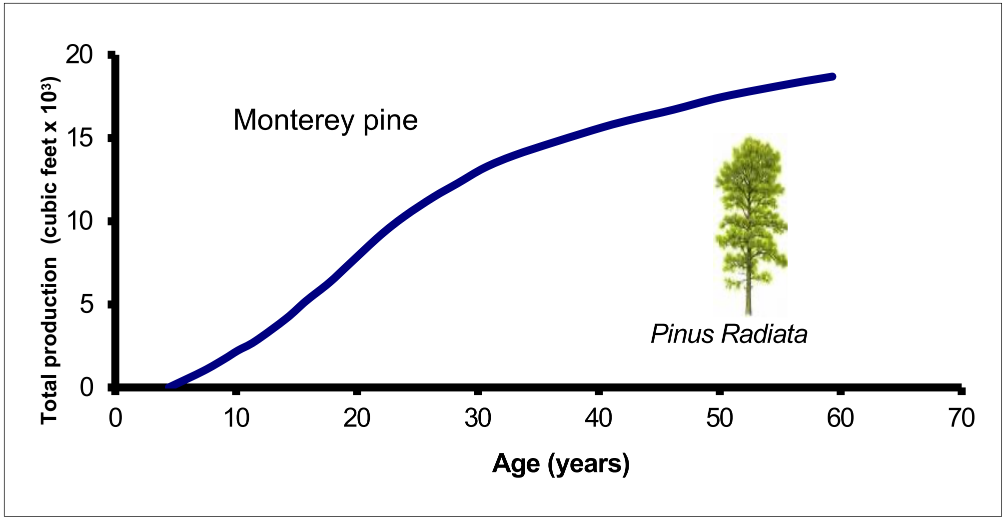
Rainbow trout

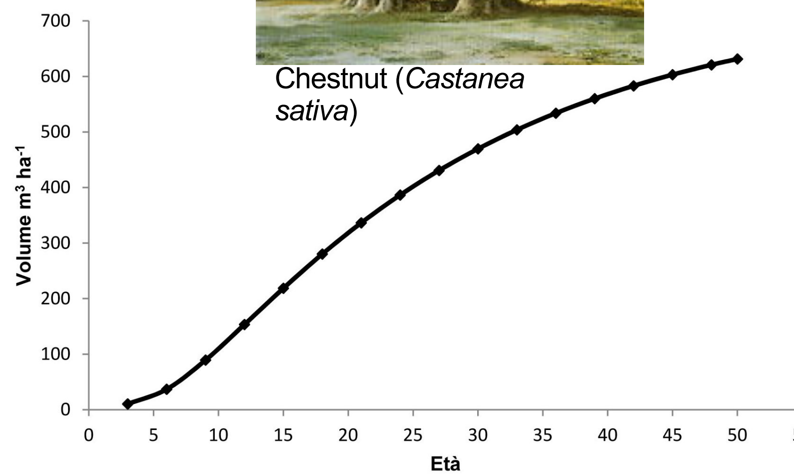
Two simple (?) problems

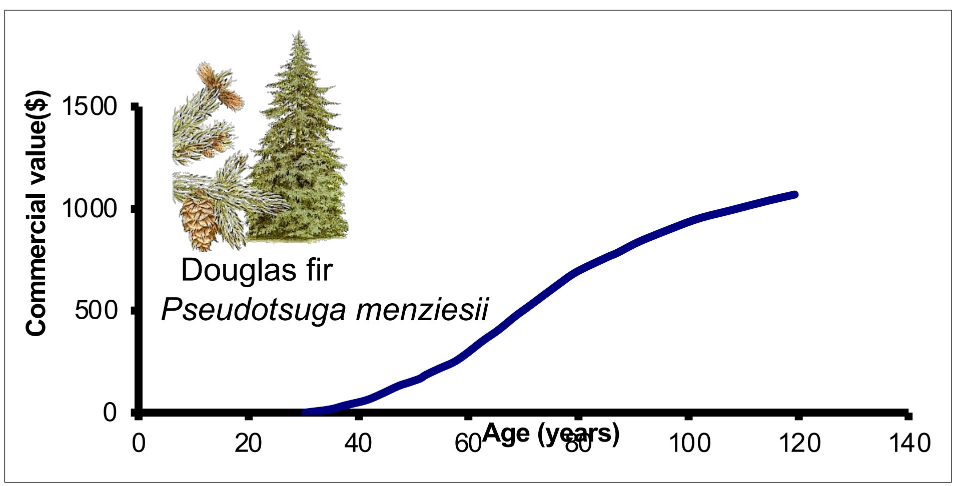
In both cases we can neglect density dependence, but account for age structure

- Optimal rotation period in the management of forests and aquaculture: finding the optimal age at which to operate harvesting
- Fish populations with a constant (or approximately constant) recruitment

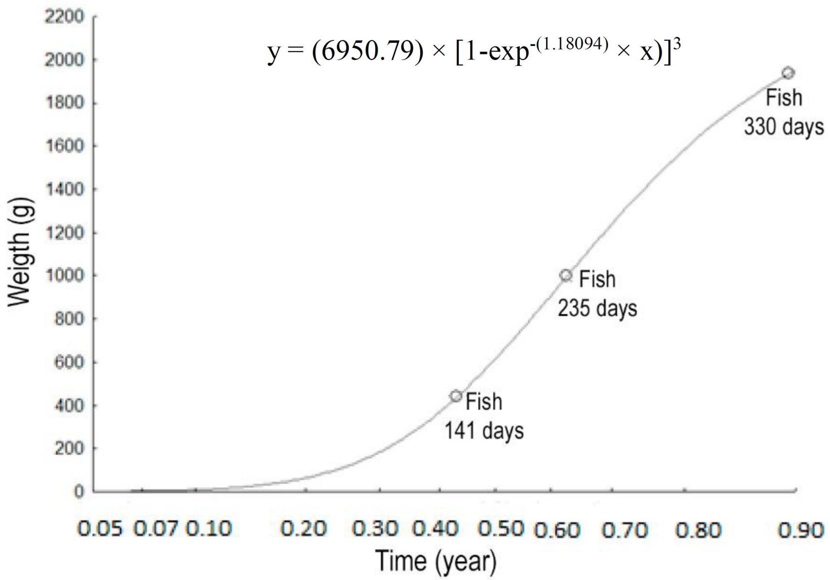
Optimal rotation period for timber production







Aquaculture



Tambaqui (*Colossoma macropomum*)

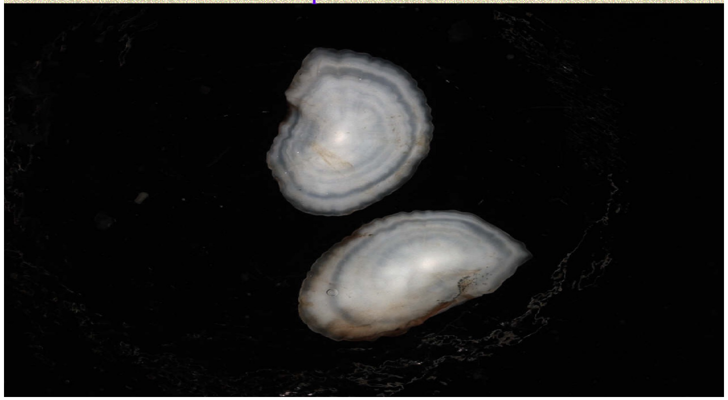
Table 4 Partial operational cost (POC) with the tambaqui biomass production in semi-dug ponds.

1. Fixed costs	**Unit**	**5 ponds**	**10 ponds**
Depreciation of facilities and infrastructure/pond (US$)	Month	78.38	63.91
Electricity/maintenance and others (US$)	Month	370.40	370.40
Labor and labor charges (70% MOD) (US$)1	Month	795.01	1,192.52
2. Variable costs			
Fingerlings (US$)	thousand	23.74	23.74
Raw protein feed 28% (US$)	US$/kg	0.427	0.427
Sole weight consumption CAA2 =1:1	US$	8,204.15	16,408.31
Mix consumption mix-2 CAA = 1.2:1	US$	9,024.57	18,049.14
Mix consumption mix-3 CAA = 1.6:1	US$	9,844.98	19,689.97

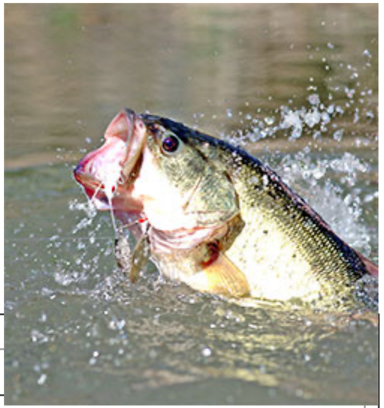
[1]MOD: Labor with wage (Brazil = US$ 233.83) with charges (70%) being one caretaker for five ponds; and one employee and one caretaker for 10 ponds.

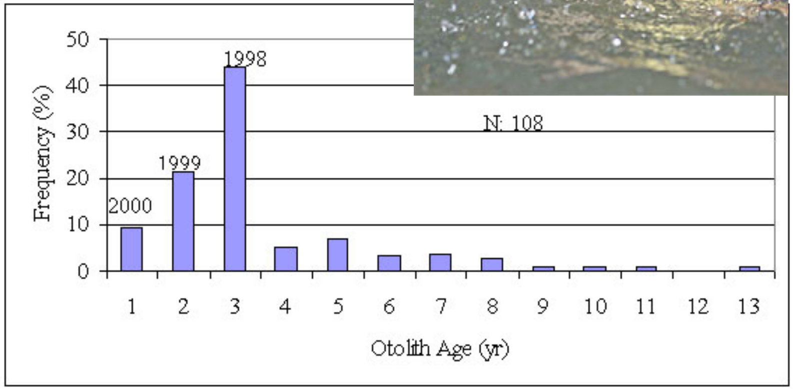
[2]CAA: Apparent food conversion. (Exchange: US$1.00 = R$ 3.72, Brazil on January 15, 2019).

Age and size structure in fish populations



Visible growth bands in otolith sections- 3year old plaice





- Estimated growth parameters from FISAT:
- Asymptotic length ($L\infty$) =26.87
- K = 0.78
- $T_{0} = 0$

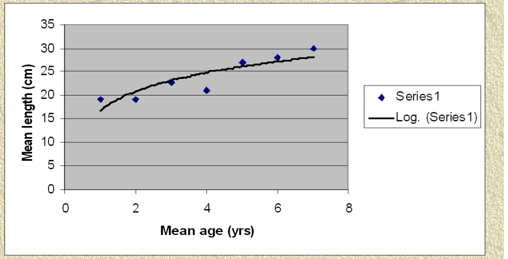


Fig. 3. Growth curve for plaice

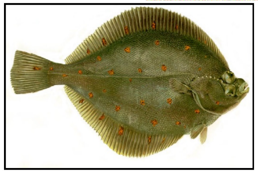
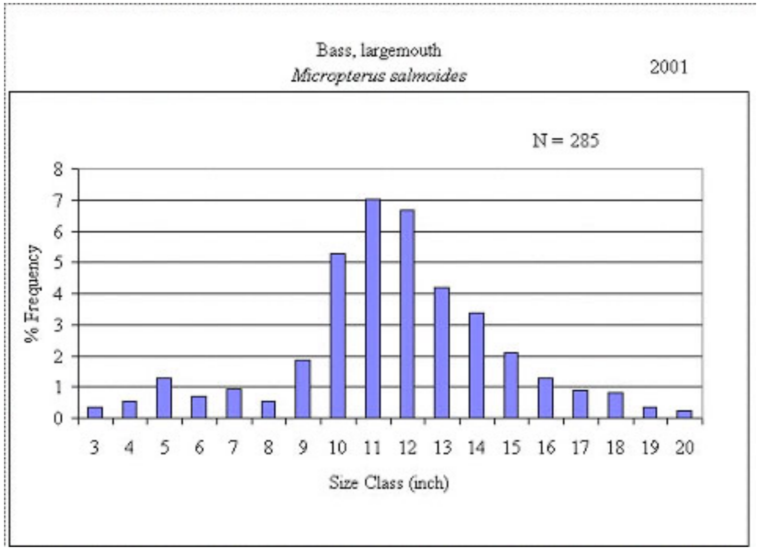
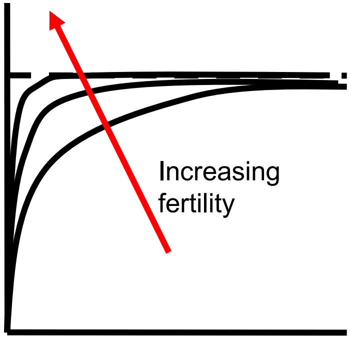


Fig 1 *Pleuronectes platessa*



Constant recruitment populations

- Recruitment is the first age class that is technically liable to capture (although fishing gear can be regulated so that it is not actually caught)
- In some populations it is independent of the parental stock
  - Recruitment is due to migration of a small fraction of juveniles to harvesting area. Reproduction takes place elsewhere (e.g. eels)
  - Strong density dependent mortality of juveniles in high fertility species (e.g. plaice). Remember Beverton-Holt model



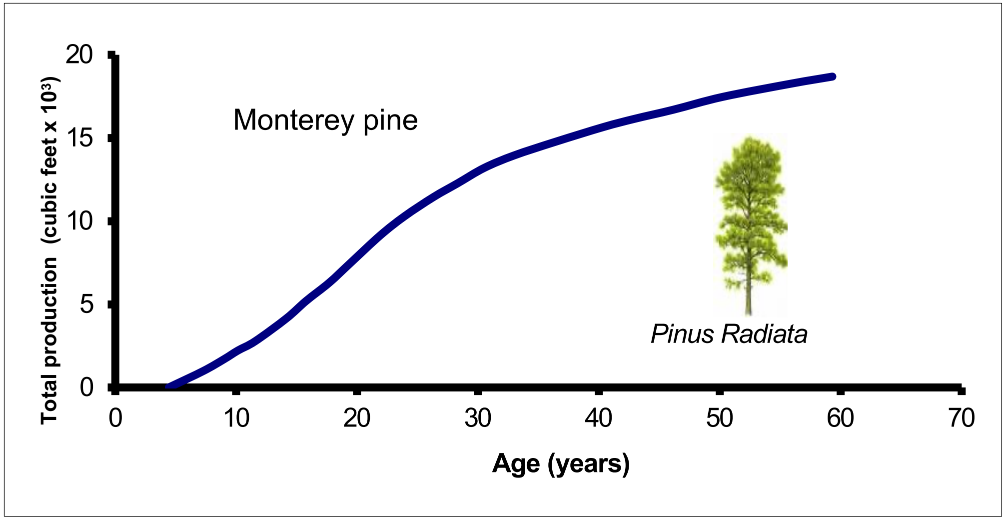
Valli di Comacchio and the eels

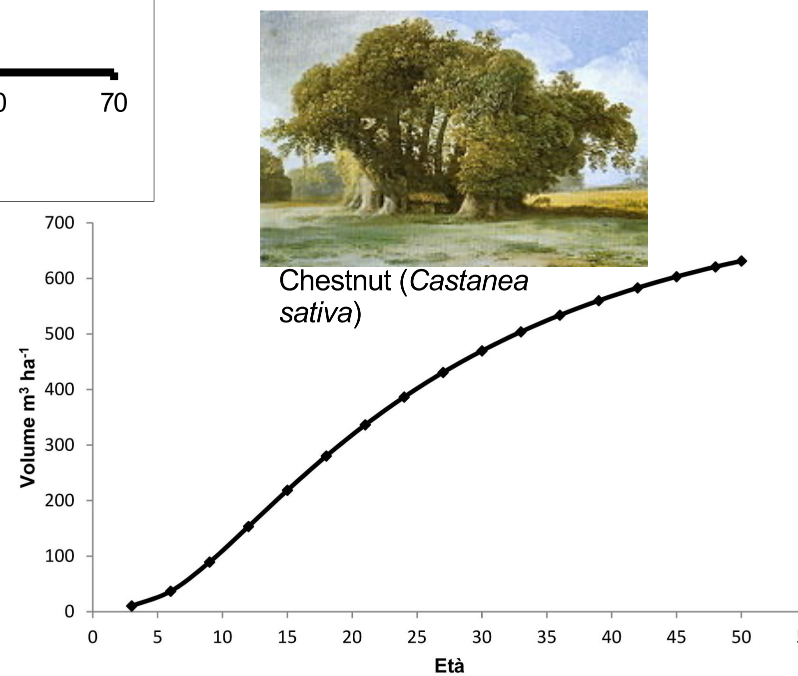
Lavoriero

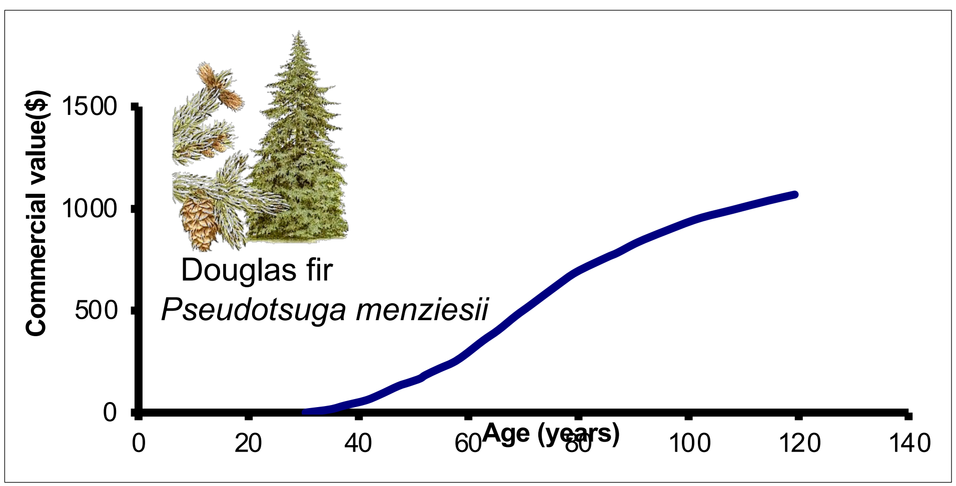
Management of age-structured fish populations in the open sea

- Analytical models aim at providing a detailed description of the population being exploited
- Age structure and body size at various ages are considered
- Simplest analytical model: constant recruitment fish populations (Beverton & Holt, 1957)

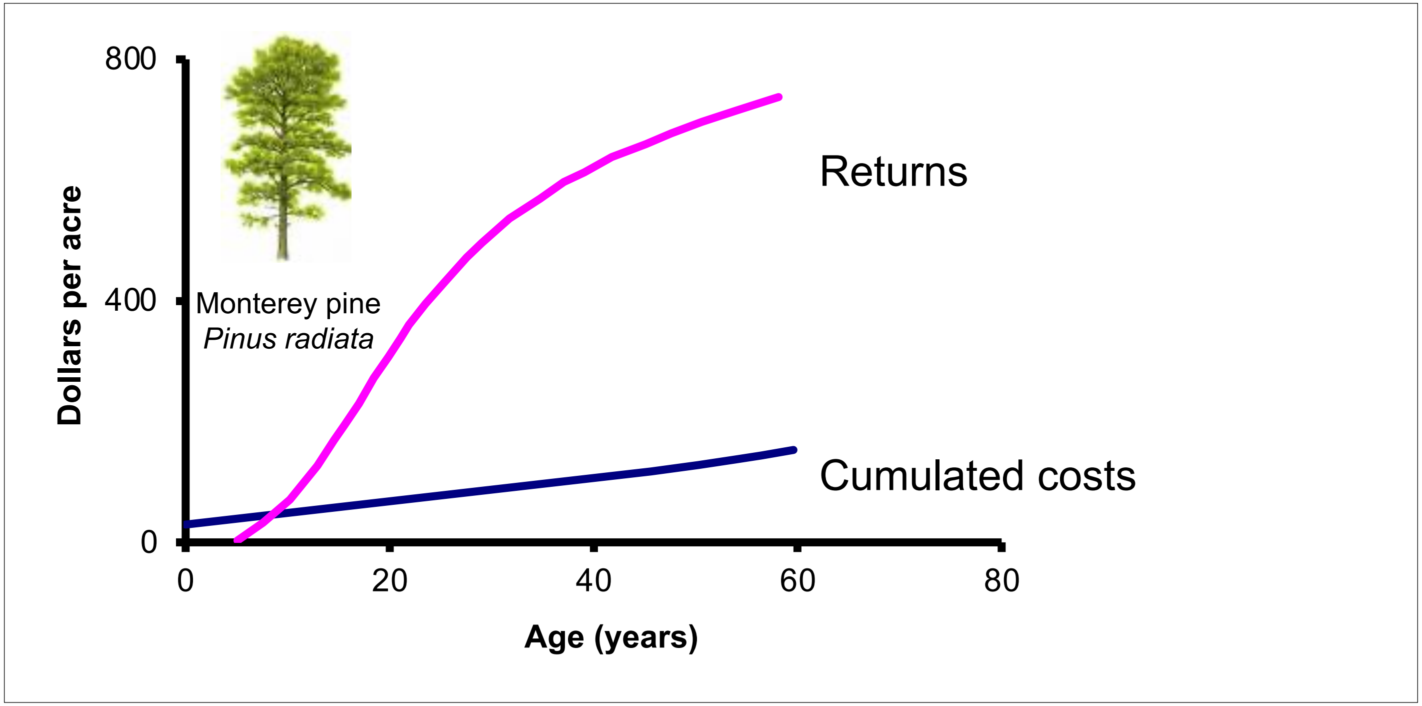
Optimal rotation period for timber production







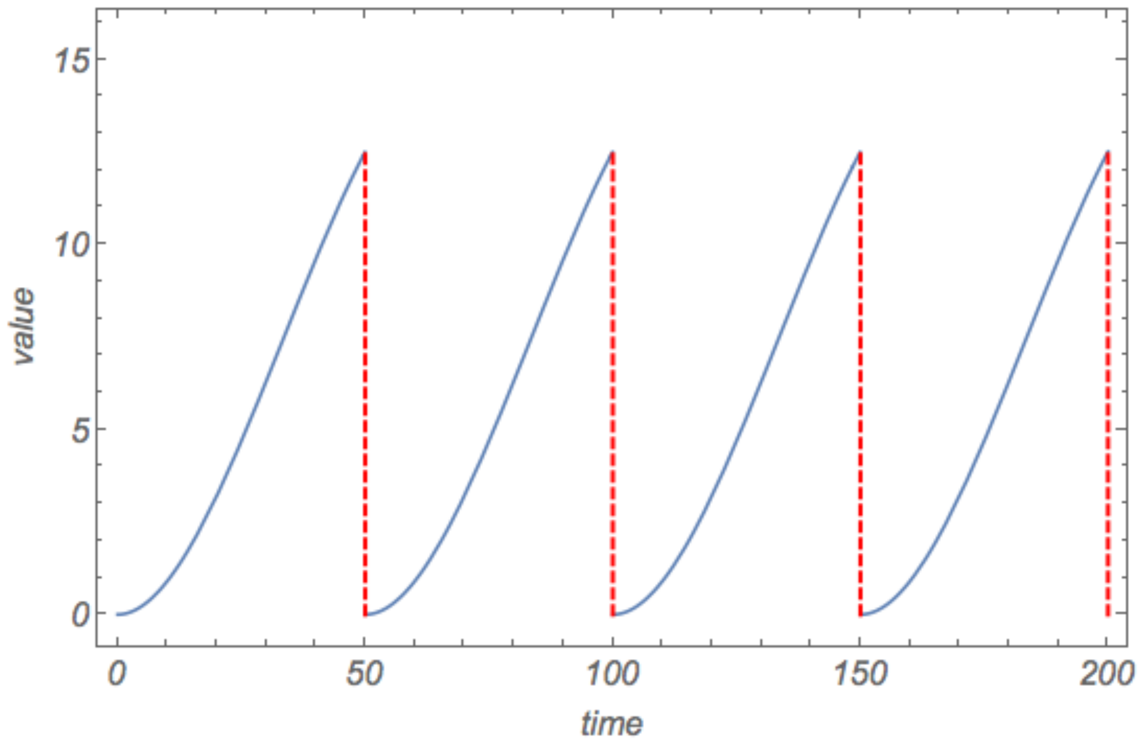
Costs and returns



$V(T)$ = commercial value of the stand at age T
$c(T)$ = cumulated planting and maintenance costs
c_A = clear-cutting cost
$\Pi_{\text{tot}} = V(T) - c(T) - c_A$ = profit

The problem of discounting

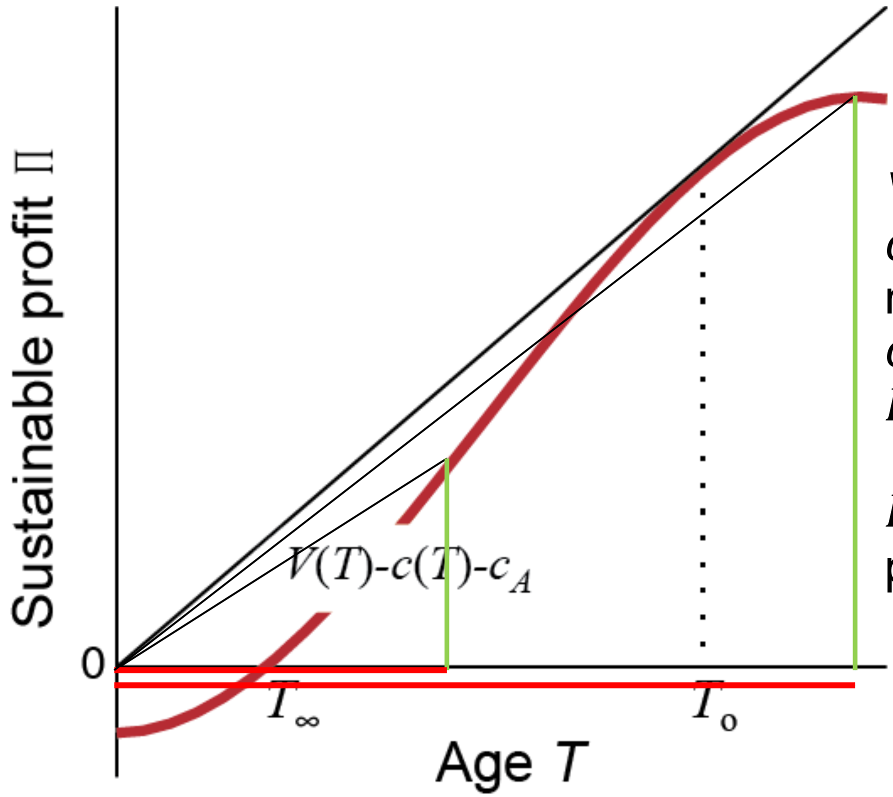
The flow of net revenues



$V(50)$ $V(50)$ $V(50)$ $V(50)$

The problem of discounting

No discounting



$V(T)$ = commercial value of the stand
$c(T)$ = cumulated planting and maintenance costs
c_A = clear-cutting cost
$\Pi_{\text{tot}} = V(T) - c(T) - c_A$ = profit

$\Pi_{\text{ave}} = (V(T) - c(T) - c_A)/T$ = average profit

Optimal period T_o which maximizes Π_{ave} satisfies

$$\frac{d\Pi_{tot}}{dT} = \Pi_{ave}$$

Average profit = marginal profit

Interest – discount rates

i = annual interest (adjusted for inflation)

Suppose a present return *PP* is invested at interest *i*

After *n* years the future return *FP* will be

$FP = PP(1+i)^n$

Thus the present value of a future return is

$PP = FP(1+i)^{-n}$

For a sequence P_0, P_1, …P_k … of profits obtained in years 0, 1,…k,…

$$PP = \sum_{j=0}^{\infty} P_j (1+i)^{-j} = \sum_{j=0}^{\infty} P_j \exp(-\delta j)$$

with $\delta = \log(1+i)$. For a continuous flow $P(t)$ of profits we have

$$PP = \int_0^{\infty} P(t)\exp(-\delta t)\,dt$$

Faustmann's formula

$$\Pi = \text{profit} = V(T) - c_A$$

Present value of a profit obtained after T years

$$V_P = \exp(-\delta T)(V(T) - c_A)$$

Present value of a flow of profits every T years

$$V_P = \sum_{k=1}^{\infty} \exp(-k\delta T)(V(T) - c_A) = \frac{V(T) - c_A}{\exp(\delta T) - 1}$$

By deriving one obtains

$$\frac{V'(T)}{V(T) - c_A} = \frac{\delta}{1 - \exp(-\delta T)}$$

Faustmann's formula

The influence of discounting

$$\frac{V'(T)}{V(T)-c_A} = \frac{\delta}{1-\exp(-\delta T)}$$

Faustmann's formula

Let the discount rate $\delta \rightarrow 0$, then

$$\lim_{\delta \to 0} \frac{\delta}{1-\exp(-\delta T)} = \frac{1}{T} \quad \rightarrow \quad V'(T) = \frac{V(T)-c_A}{T}$$

Average profit = marginal profit

Let the discount rate $\delta \rightarrow \infty$, then

$$\lim_{\delta \to \infty} \frac{\delta}{1-\exp(-\delta T)} = \infty \quad \rightarrow \quad V(T)-c_A = 0$$

Rent dissipation
Bionomic equilibrium

Douglas fir

Age (years)	Net stumpage value ($)	Average profit ($/yr)
30	0	0
40	43	1.08
50	143	2.86
60	303	5.05
70	497	7.1
80	650	8.12
90	805	8.94
100	913	9.13
110	1000	9.09
120	1075	8.93

Annual interest rate *i* (%)	Optimal rotation period (years)	Annual discounted profit ($/yr)
0	100	9.1
3	70	7.1
5	63	5.6
7	56	4.2
10	49	2.8
15	43	1.7
20	40	1.2

The fin whale (*Balaenoptera physalus*)

Logistic model with $K = 400{,}000$ $r = 0.08\ yr^{-1}$

Optimization of net monetary benefit	
Discount rate δ (% $year^{-1}$)	Optimal population x^*
0	220000
1	200000
3	163000
5	133000
10	86000
15	67000
20	59000
$+\infty$	40000

Current population estimate = 47,300