Problems on the Management of Renewable Resource Harvesting
Solutions (detailed in some cases, just the results in other cases)

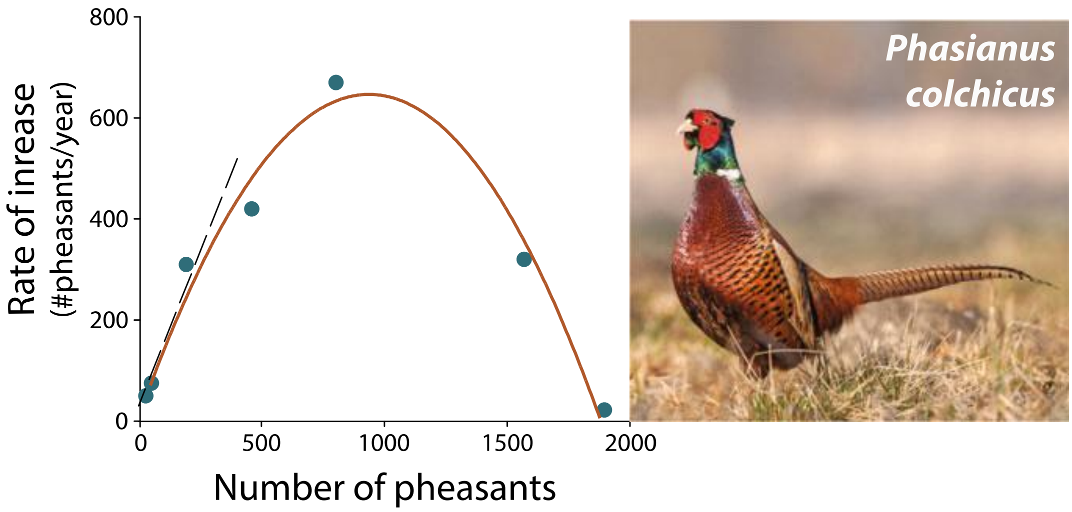
Problem M1

The common pheasant (*Phasianus colchicus*) was introduced into Protection Island in 1937. By following the demographic growth of this bird along time it has been possible to obtain the relationship linking the population rate of increase to the population size, as shown in the figure.

Assume that pheasant hunting is permitted and the regulation policy is based on granting licenses. You know that

- effort is measured as No. of operating hunters
- the catchability coefficient is 0.01 No. hunters^{-1} year^{-1}
- only half of licensed hunters is actually hunting in the average.

Calculate the number L of licenses to be granted that guarantees the maximum sustainable yield and the corresponding pheasant population size at equilibrium. Finally calculate the effort E_{ext} that would lead population to extinction.



Problem M1 The rate of increase of the common pheasant (*Phasianus colchicus*) in Protection Island

Solution

Let x be the number of pheasants. From the graph it is clear that the rate of increase dx/dt is a parabolic function of x. Therefore, the population dynamics is well described by a logistic model

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)$$

The value of K can be found by estimating the number (different from zero) of pheasants where $dx/dt = 0$, namely $K = 1850$. Also, the per capita rate of increase r is the slope of the tangent to the parabola in $x = 0$, that is $r = 380/250 = 1.52$ year^{-1}.

The dynamics of hunted pheasants can be thus described by the Schaefer model

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - qEx = rx\left(1 - \frac{x}{K}\right) - q\frac{L}{2}x$$

where E is the effort, which is equal to $L/2$. The stable equilibrium x_{eq} corresponding to the number L of licenses is thus

$$x_{eq} = K\left(1 - \frac{q}{r}\frac{L}{2}\right)$$

and the sustainable yield is

$$Y = qK\frac{L}{2}\left(1 - \frac{q}{r}\frac{L}{2}\right)$$

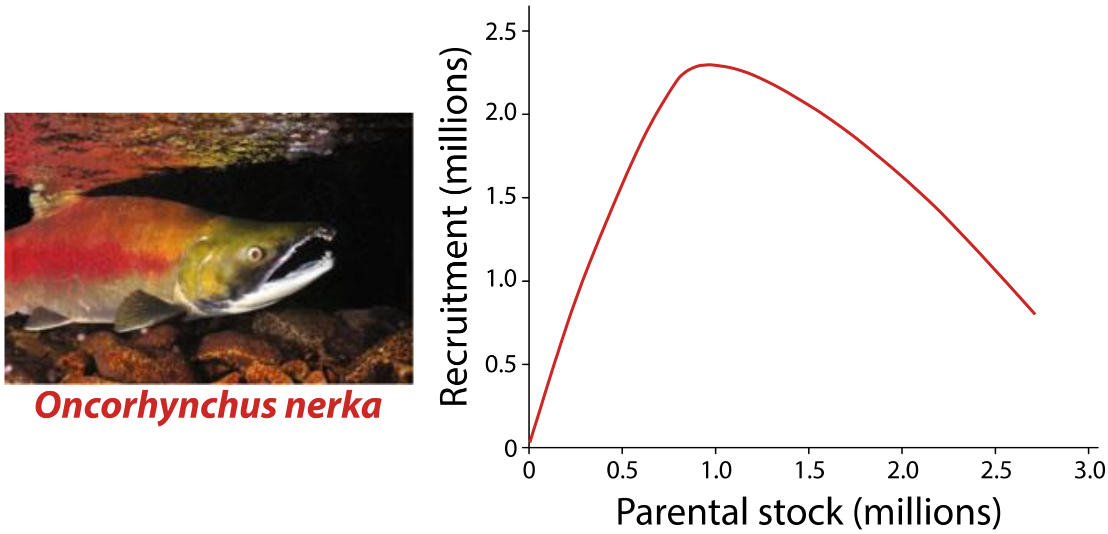
The MSY can be found by imposing $dY/dL = 0$, that is

$$q\frac{K}{2}\left(1 - q\frac{L}{r}\right) = 0 \rightarrow L = \frac{r}{q}$$

From which $L_{MSY} = 1.52/0.01 = 152$ licenses.
The number of licenses L_{ext} that would bring the population to extinction is easily found by setting $x_{eq} = 0$, thus finding $L_{ext} = 2r/q = 304$ licenses corresponding to an effort of 152 operating hunters.

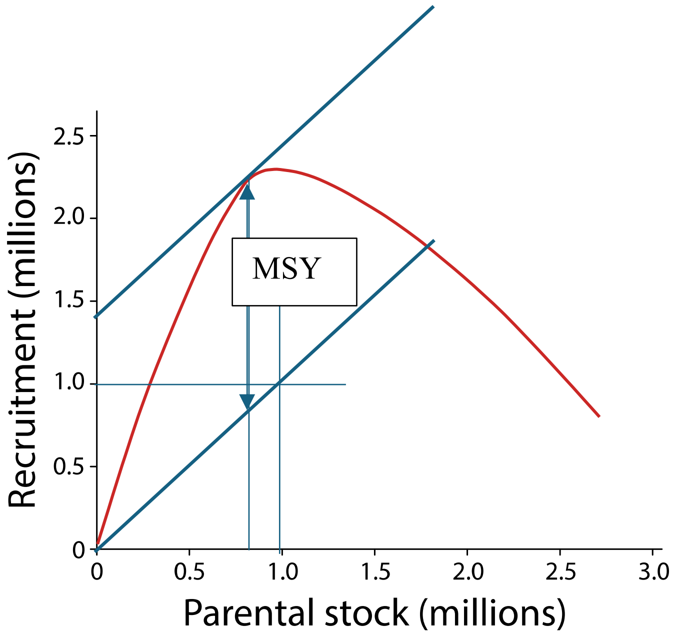
Problem M2

The graph shown in the figure reports the stock-recruitment relationship for the sockeye salmon (*Oncorhynchus nerka*) of river Skeena (British Columbia, Canada). Approximately determine the maximum sustainable yield (as million fish).



Problem M2 The stock-recruitment relationship for sockeye salmon (*Oncorhynchus nerka*) in river Skeena

Solution
Let P the parental stock and $F(P)$ the stock-recruitment relationship shown in the graph. The solution can be found graphically by remembering that the stock at equilibrium P_{eq} providing the MSY is the one such that $Y = F(P) - P$ is maximum. This corresponds to finding where $dF/dP = 1$. Graphically this means finding the tangent to the stock-recruitment curve that is parallel to the 45° degrees straight line.



Therefore P_{MSY} = 0.8 million salmon and MSY = $F(P_{MSY}) - P_{MSY}$ = 2.3 million – 0.8 million = 1.5 million salmon.

Problem M3

The northern prawn fishery is one of the most important Australian fisheries. The tiger prawns *Penaeus esculentus* and *Penaeus semisulcatus* are two most prominent species subject to harvesting. Wang and Die (1996) determined the stock-recruitment curves for the two species from available data.

If B_k is the total biomass (tonnes) in year k, we can approximate the curves by a Beverton-Holt model as follows:

$$B_{k+1} = \lambda B_k \big/ (1 + \alpha B_k)$$

where

- for *Penaeus esculentus*: $\lambda = 1.6$ and $\alpha = 0.00019$;
- for *Penaeus semisulcatus*: $\lambda = 2.16$ and $\alpha = 0.0004$.

The effort is measured as boat-days (that is the number of operating boats times the days they are operating in a given year). From the estimates of the two researchers we can derive the catchability coefficients for the two prawns:

- *Penaeus esculentus*: $q = 3 \times 10^{-4}$ (boat-days)$^{-1}$;
- *Penaeus semisulcatus*: $q = 3.5 \times 10^{-4}$ (boat-days)$^{-1}$.

For each of the two prawns, determine:

(a) the Maximum Sustainable Yield (MSY);
(b) the effort that provides the MSY;
(c) the corresponding prawn biomass.

Solution

Let $F(B)$ be the stock-recruitment function, that is $F(B) = \frac{\lambda B}{1+\alpha B}$. As in the previous exercise the biomass B_{MSY} that corresponds to the MSY is the one such that $dF/dB = 1$, that is

$$\frac{\lambda}{(1+\alpha B)^2} = 1$$

from which we get $B_{MSY} = \frac{\sqrt{\lambda}-1}{\alpha}$. On the other hand, the dynamics of the exploited stocks is given by the equation

$$B_{k+1} = \frac{\lambda B_k}{1+\alpha B_k} exp(-qE_k)$$

Therefore, at the equilibrium corresponding to MSY we obtain

$$B_{MSY} = \frac{\lambda B_{MSY}}{1+\alpha B_{MSY}} exp(-qE_{MSY})$$

from which it is possible to derive E_{MSY} as a function of B_{MSY}, namely

$$E_{MSY} = \frac{1}{q} ln\left(\frac{\lambda B_{MSY}}{1+\alpha B_{MSY}}\right)$$

Finally, the MSY can be derived from B_{MSY} as the difference between the recruitment and the stock, that is MSY = $\frac{\lambda B_{MSY}}{1+\alpha B_{MSY}} - B_{MSY}$.

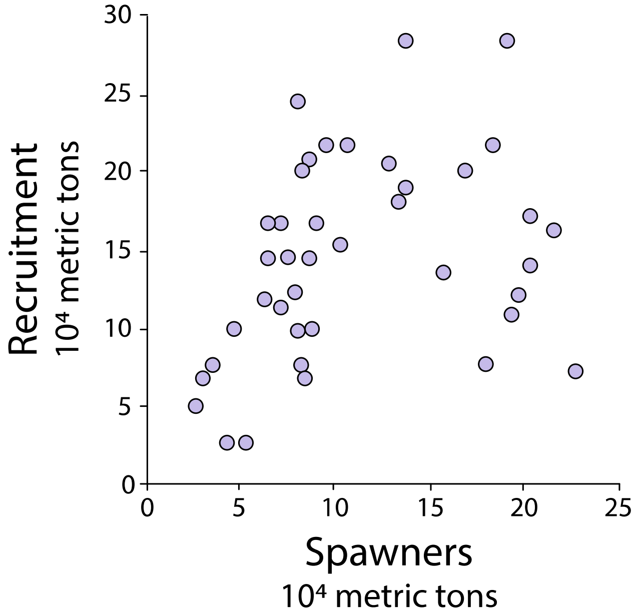
Substituting the numerical values for the two prawns we obtain

(a) $MSY_{esculentus}$ = 369.36 tonnes $MSY_{semisulcatus}$ = 551.53 tonnes
(b) E_{MSYesc} = 783.34 boat-days $E_{MSYsemi}$ = 1100.15 boat-days
(c) B_{MSYesc} = 1394.27 tonnes $B_{MSYsemi}$ = 1174.23 tonnes

Problem M4

Zhang et al. (2012) describe the dynamics of the Korean stock of Japanese horse mackerel (*Trachurus japonicus*). From data, as shown in the Figure, they have obtained a stock-recruitment relationship linking the biomass (in metric tons) of one generation (the spawners B_t) to the biomass of the subsequent generation (the recruitment B_{t+1}).

Problem M4 Stock-Recruitment relationship for the Japanese horse mackerel (*Trachurus japonicus*) as emerging from data (1970–2009, dots) and estimated via Beverton-Holt (black dotted) and Ricker-like (solid gray) curves. Redrawn after Zhang et al. (2012)



$$B_{t+1} = A(1 - \exp(-\lambda B_t))$$

with $A = 1.9 \times 10^5$ tonnes and $\lambda = 1.42 \times 10^{-5}$ tonnes^{-1}.

You are required to:

(a) Find the constant escapement policy that maximizes the sustainable yield of the mackerel stock;
(b) Calculate the corresponding MSY;
(c) Calculate the fraction u of the recruitment that is taken by means of the optimal policy at equilibrium.

Finally, assume that effort is measured in number of operating vessels, that the catchability coefficient is $q = 0.02$ (No. of vessels)$^{-1}$ year^{-1} and that there are 150 fishing vessels that operate during a fishing season of duration T (months); how long should T be for actually harvesting the fraction u?

Solution
Solution very similar to previous exercise. Here are the results:
The optimal constant escapement is $B_{MSY} = \frac{ln(\lambda A)}{\lambda}$ =69,895 tonnes in each generation.
The corresponding MSY is 49,682 tonnes.
The fraction of the recruitment harvested at MSY is $u_{MSY} = 0.415$.
The fishing season is $T_{MSY} = 0.179$ years = 2.15 months.

Problem M5

Differently from Pacific salmon, the Atlantic salmon (*Salmo salar*) can reproduce several times. In other words the survival of reproductive adults is not zero.

For the small Norwegian Imsa River the relationship between the adults (measured as tonnes of biomass) returning for the first time to the river (A) in order to reproduce is approximately linked to the total biomass (tonnes) of adults (N) by the following relationship (Jonsson et al. 1998) between year k and year $k+1$:

$$A_{k+1} = \frac{\lambda N_k}{(1+\alpha N_k)}$$

Problem M5 The Atlantic salmon

with $\lambda = 2500$ and $\alpha = 24$ tonne^{-1}. As some salmon survive after reproduction the total biomass of adults in year $k+1$ is given by the sum of A_{k+1} plus the fraction s of adults N_k that survive to the next year. Assume $s = 0.25$.

Determine the constant escapement policy that provides the Maximum Sustainable Yield (measured as number of harvested salmon).

Solution

The equation for the dynamics of N_k is

$$N_{k+1} = \frac{\lambda N_k}{1+\alpha N_k} + sN_k = F(N_k)$$

As in the previous exercises one can easily find the optimal escapement providing the MSY

$$N_{MSY} = \frac{1}{\alpha}\left(\sqrt{\frac{\lambda}{1-s}} - 1\right) = 2.36 \text{ tonnes.}$$

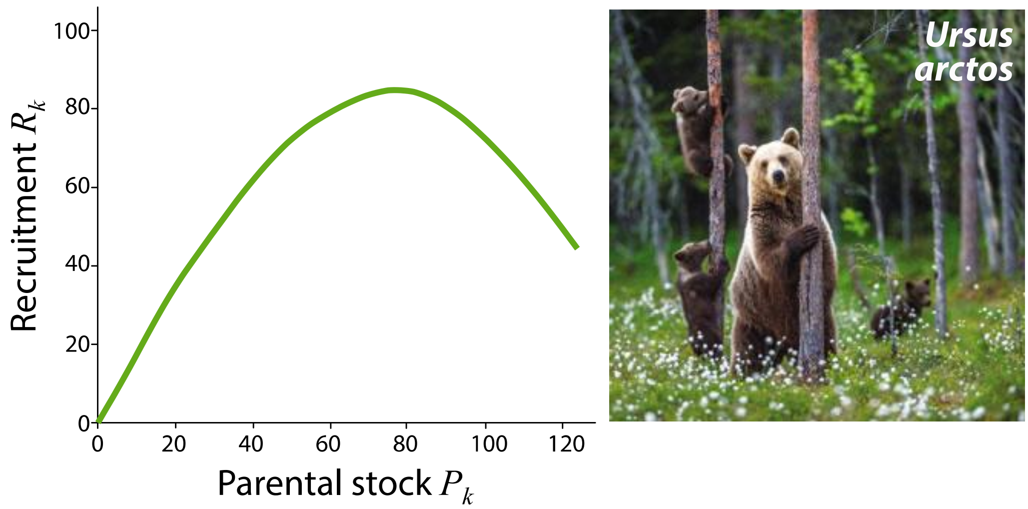
Problem M6

McCullough (1981) studied the dynamics of grizzly bears (*Ursus arctos*) in the Yellowstone Park. He reports the stock-recruitment curve shown in the Figure, which links the number P_k of adult bears in year k (namely, the individuals that are 4-years-old or older in year k) to the recruitment (R_k), namely the number of adult bears four years later.

Assume that the Park management allows hunting of adult grizzlies according to the following policy

$$H_k = \begin{cases} 0 & \text{if } R_k \leq 35 \\ 0.5(R_k - 35) & \text{if } R_k > 35 \end{cases}$$

so that a minimal recruitment of 35 adult bears is guaranteed. Evaluate the implications of implementing this policy on the population dynamics.



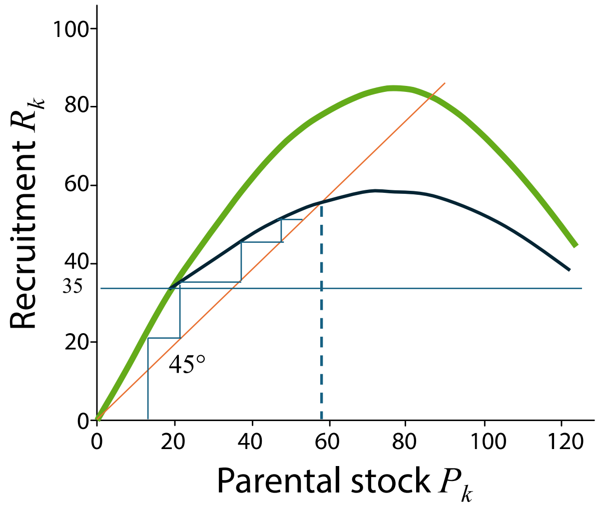
Problem M6 The stock-recruitment relationship for grizzly bears in the Yellowstone Park

Solution

The solution is found graphically by implementing the policy on the figure below: the black curve shows the modification of the stock recruitment function $F(P)$ to take into account the dynamics with harvesting, namely

$$P_{k+1} = \begin{cases} F(P_k) & \text{if } R_k \leq 35 \\ F(P_k) - 0.5(F(P_k) - 35) & \text{if } R_k > 35 \end{cases}$$

From the figure below one can see that the intersection of the modified curve with the 45° line provides the equilibrium of 58 bears, which is well above the minimal requirement of a recruitment of 35 bears. This equilibrium is stable as can be easily determined by plotting the corresponding staircase diagram.



Problem M7

There has been an international debate in the past regarding the opportunity of catching Antarctic krill (*Euphausia superba*), a small crustacean that forms huge swarms in the waters surrounding Antarctica and is a fundamental food for many baleen whales. As the whale stocks have collapsed (and their recovery following the international moratorium on whale hunting will take decades), some researchers proposed that krill might be fished without impairing the functioning of the Antarctic ecosystem.

Discuss the problem by writing a simple prey-predator (krill biomass-whale biomass) of the Lotka-Volterra kind, in which the prey grows in a logistic way. Assume that krill is fished with a constant effort E, while hunting of whales does not take place. Determine the stable equilibria of the prey-predator system while the parameter E varies. Find out how the biomass of the sustainable krill catch varies for increasing E.

Problem M7 The Atlantic krill

Solution

Introduce the variables x = krill biomass, y = whale biomass. The corresponding Lotka-Volterra model with exploitation of the krill biomass is as follows

$$\frac{dx}{dt} = rx(1 - x/K) - qEx - pxy$$
$$\frac{dy}{dt} = epxy - my$$

where r is the intrinsic rate of increase and K the carrying capacity of the krill, q is the catchability coefficient, p the predation rate coefficient, e the krill-to-whale metabolic efficiency, m the whale death rate.

The stable equilibria of the system can be determined by using the isocline method. We graph the loci where $dx/dt = 0$ (prey isocline, red) and $dy/dt = 0$ (predator isocline, green).

Prey isocline

$x[r(1 - x/K) - qE - py] = 0$

$x = 0$ trivial isocline

$r\left(1 - \frac{x}{K}\right) - qE - py = 0$ nontrivial isocline from which

$y = (r - qE)/p - rx/pK = 0$

Predator isocline

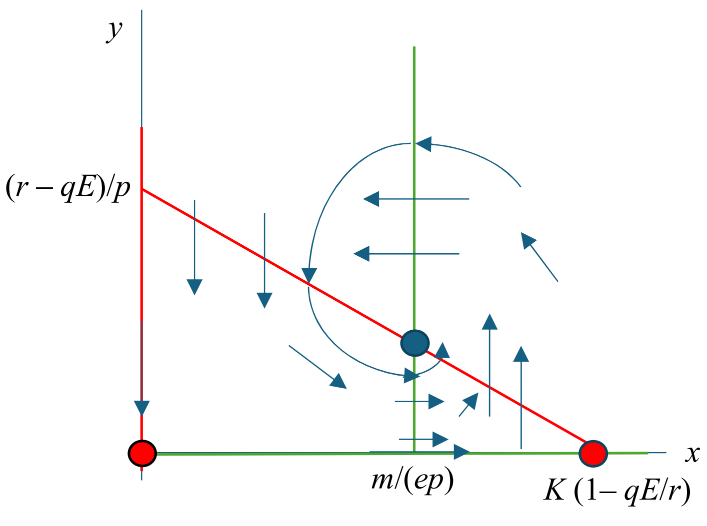
$epxy - my = 0$

$y = 0$ trivial isocline

$x = m/(ep)$ non trivial isocline

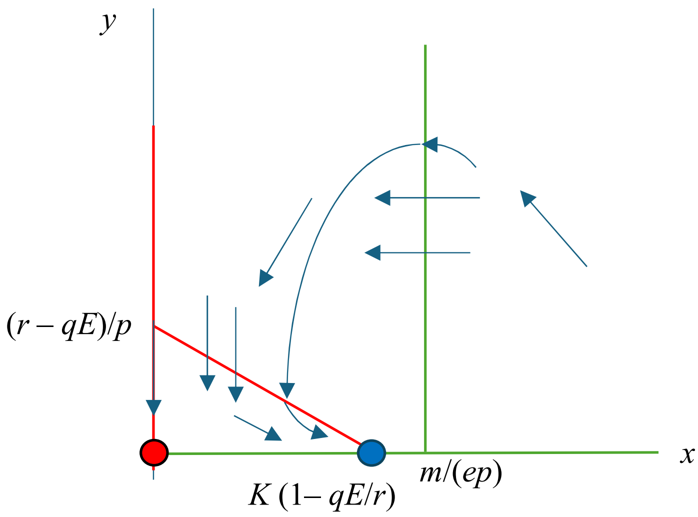
We graph the loci where $dx/dt = 0$ (prey isocline, red) and $dy/dt = 0$ (predator isocline, green). There are two possible situations:

1) $K(1 - qE/r) > m/(ep)$, that is the effort E is lower than $\frac{r}{q}(1 - \frac{m}{epK})$. In this case the two nontrivial isoclines intersect and the corresponding equilibrium $[x_{eq} = m/(ep),\ y_{eq} = \frac{r}{p}\left(1 - \frac{m}{epK}\right) - qE]$ is stable (see figure below which shows the possible trajectory directions, the stable equilibrium in blue and the two other unstable equilibria in red)



2) $K(1 - qE/r) > m/(ep)$, that is the effort E is larger than $\frac{r}{q}(1 - \frac{m}{epK})$. In this case the two nontrivial isoclines do not intersect, while the nontrivial predator isocline intersects the trivial prey isocline

and the stable equilibrium is the one with whale biomass $y_{eq} = 0$ (blue dot) and the krill biomass $x_{eq} = K\,(1 - qE/r)$.



Therefore, we can conclude that increasing the fishing effort on krill leads to the extinction of the whales. As for the sustainable catch Y of krill as a function of the effort E we obtain

$$Y = qEx_{eq} = \begin{cases} \frac{qm}{ep}E \quad \text{if } E < \frac{r}{q}(1 - \frac{m}{epK}) \\ qKE\left(1 - \frac{qE}{r}\right) \text{ if } \frac{r}{q}\left(1 - \frac{m}{epK}\right) \leq E < r/q \\ 0 \quad \text{if } E > r/q \end{cases}$$

Problem M8

Assume you want to rationally regulate deer hunting in a grassland where the animals can extensively graze. To that end, describe the resource (grass)—consumer (deer) system dynamics by the equations

$$\frac{dG}{dt} = w - d_G G - pGD$$

$$\frac{dD}{dt} = -d_D D + epGD - uD$$

where G and D, respectively, indicate the biomass of grass and of deer (tonnes) and u is the mortality rate due to deer hunting. You know that

- w = net primary production = 100 tonnes year^{-1}
- d_G = grass death rate = 10 year^{-1}
- p = grazing rate coefficient = 0.2 tonnes^{-1} year^{-1}
- e = conversion coefficient = 0.1
- d_D = deer death rate = 0.1 year^{-1} .

Calculate the sustainable yield of deer biomass as a function of hunting mortality u. Then find the maximum sustainable yield.

Solution
As in the previous exercise, use isoclines $dG/dt = 0$ and $dD/dt = 0$.
The sustainable yield Y as a function of u is

$$Y = \begin{cases} u\left(\dfrac{10}{0.1+u} - 50\right) & \text{if } u < 0.1 \\ 0 & \text{if } u \geq 0.1 \end{cases}$$

The value of u corresponding to the MSY is
$u_{MSY} = 0.041$ year^{-1} and MSY = 0.858 tonne year^{-1}.

Problem M9

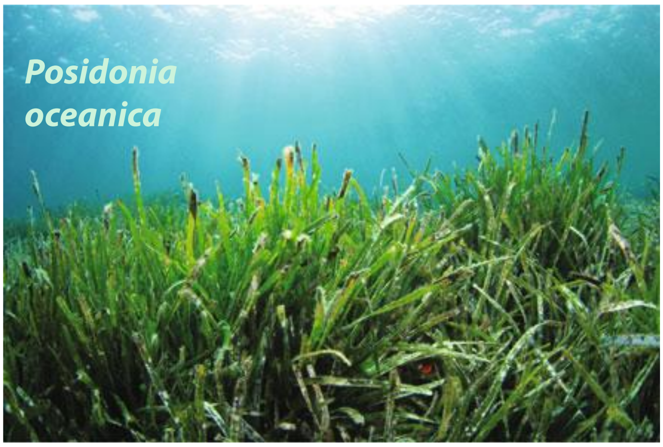
Trawling is a major disturbance to fish habitat. In particular beds of *Posidonia oceanica*, an ecologically important seagrass, are badly impacted by this kind of fishing. The effect is noxious to the fishery itself because the fish carrying capacity (and thus the fish availability to the fishermen) is of course an increasing function of the remaining habitat. Thus if seagrass is destroyed the fishery yield decreases considerably.

To analyse the problem write a model that describes the dynamics of the fish stock biomass B (kg m^{-2}) and the *Posidonia* biomass P (kg m^{-2}). The dynamics of B is logistic, however the carrying capacity is not constant but increases with *Posidonia* biomass P. Fish biomass is harvested at a rate qEB where the effort E is measured as number of operating trawlers. The dynamics of *Posidonia* biomass P is described by a constant recruitment w of new seagrass biomass (kg m^{-2} year^{-1}) and a constant mortality rate m (year^{-1}). When the trawlers are operating, part of the *Posidonia* biomass is removed at a rate zEP. The parameter values are as follows:

- r = instantaneous rate of fish increase = 0.1 year^{-1};
- K = fish carrying capacity (kg m^{-2}) = kP where $k = 0.05$;
- $q = 0.01$ (No. of trawlers)$^{-1}$ year^{-1};
- $w = 0.9$ kg m^{-2} year^{-1};
- $m = 1.8$ year^{-1};
- $z = 0.05$ (No. of trawlers)$^{-1}$ year^{-1}.

Find out the effort (number of trawlers) that maximizes the sustainable yield of fish biomass. Calculate the corresponding standing biomass of fish and *Posidonia* and the MSY.

Problem M9 The habitat created by *Posidonia oceanica*



Solution

The dynamics of *Posidonia* biomass and fish stock biomass is provided by the equations

$$\frac{dP}{dt} = w - mP - zEP$$

$$\frac{dB}{dt} = rB\left(1 - \frac{B}{kP}\right) - qEB$$

By doing the appropriate calculations one finds the effort E_{MSY} corresponding to MSY, namely $E_{MSY} = 4.69 \sim 5$ operating vessels.

Considering an effort of 5 operating vessels, one obtains $B_{MSY} = 0.011$ kg m^2, $P_{MSY} = 0.44$ kg m^2, MSY = 0.00055 kg m^2 year^{-1}.

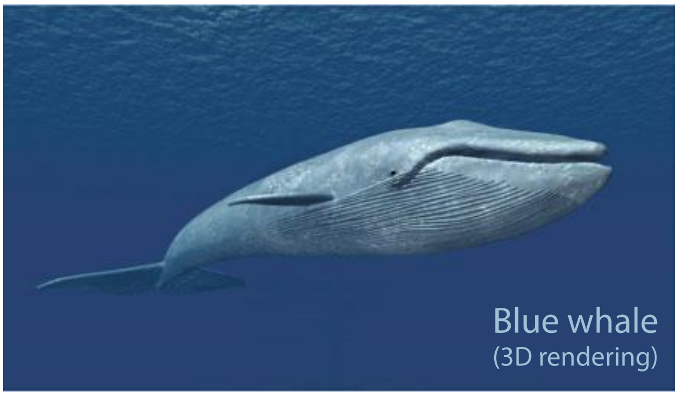
Problem M10

The whales of the genus *Balaenoptera* have been severely depleted by the extensive hunting carried out during the twentieth century. A very rough way of measuring their total biomass is the Blue Whale Unit (BWU), adopted by the International Whaling Commission which equated two fin whales and six Sei whales to one blue whale. For the complex of these whales one can use a Schaefer model (logistic growth of whales and harvesting rate proportional to the product of effort and whale stock). The effort was measured as the total number of hunting days per year. Assume the following parameters

- K = carrying capacity = 400,000 BWU
- r = intrinsic instantaneous rate of increase = 0.05 year^{-1}
- q = catchability coefficient = 1.3×10^{-5} (hunting days)$^{-1}$

and calculate the bionomic equilibrium under the hypothesis that the opportunity cost of one hunting day is €5,000 and the selling price of one BWU is €75,000.

Problem M10 A 3D rendering of a blue whale



Solution

It is well-known that the biomass N_B at the bionomic equilibrium is equal to $c/(pq)$ where c is the cost of one unit of effort and p the selling price of one unit of biomass. Therefore N_B = 5,128 BWU, very close to near extinction.

The Schaefer model is

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - qEN$$

and thus the effort E_B at the bionomic equilibrium is $E_B = \frac{r}{q}\left(1 - \frac{N_B}{K}\right)$ = 3,797 hunting days. The corresponding yield is $Y_B = q\, E_B\, N_B$ = 253 BWU year^{-1}.

Problem M11

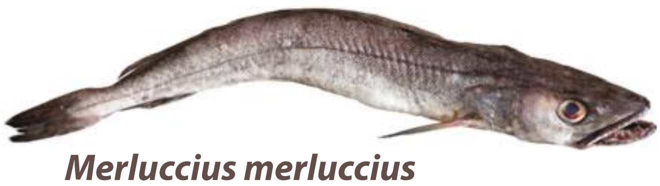
The dynamics of the stock of the European hake (*Merluccius merluccius*) of the Adriatic sea can be described by the logistic model. Levi and Giannetti (1973) estimated the following demographic parameters:

- K = carrying capacity = 5,000 tonnes
- r = intrinsic instantaneous rate of increase = 1.7 year^{-1}.

They also used the tonnes of consumed fuel per year as a measure of effort and estimated the catchability coefficient as $q = 2 \times 10^{-5}$ $(\text{fuel tonne})^{-1}$.

Assume that the selling price of hake is €6,000 per hake tonne, while the opportunity cost is c = €140 per fuel tonne. Find the values of the hake stock and the effort at bionomic equilibrium. Then calculate the effort that would guarantee the maximum sustainable profit and the corresponding stock, catch and profit.

Problem M11 The European hake



Solution

The calculation of the bionomic equilibrium follows the same steps of the previous exercise. Therefore, we have

N_B = 1,166.67 hake tonnes, E_B = 65,166.67 fuel tonnes, Y_B = 1,520.56 hake tonnes year^{-1}.

To find the effort that maximizes the sustainable profit $P(E) = pY - cE$, we must first consider that in the logistic model the biomass at equilibrium corresponding to a given effort is

$$N_{eq} = K\left(1 - \frac{q}{r}E\right)$$

and therefore

$$P(E) = pqKE\left(1 - \frac{q}{r}E\right) - cE$$

Then the effort that maximizes the sustainable profit is given by

E_{opt} = 32,583.33 fuel tonnes.

The corresponding stock is N_{opt} = 3,083.33 tonnes, catch is Y_{opt} = 2,009.31 hake tonnes year^{-1}, profit P_{opt} = 7,494,167 € year^{-1}.

Problem M12

The Pacific halibut (*Hippoglossus stenolepis*) is a large bottom-feeding fish that is harvested in the North Pacific. The fishing vessels use gears called skates. A halibut skate is a long rope to which hooks are attached every ten yards or so. Effort can be thus measured as number of deployed skates.

Clark (1976) reports the following values for a logistic model describing the halibut population:

- intrinsic instantaneous rate of demographic increase $r = 0.71$ year^{-1};
- carrying capacity $K = 80.5 \times 10^6$ kg.

Before 1930 the fishery was not regulated, so we may assume that the bionomic equilibrium had been reached, namely $N_B = 17.5 \times 10^6$ kg. It is also estimated that the catchability coefficient is $q = 10^{-3}$ (No. of skates)$^{-1}$ year^{-1}.

Let c be the opportunity cost of operating one skate per year and p the selling price of one kg of halibut. From this information derive:

(i) the ratio c/p,
(ii) the optimal effort that maximizes the total profit of the fishery, and
(iii) the corresponding annual yield in kg.

Problem M12 The Pacific halibut

Solution
c/p = 17,500 kg (No. of skates)$^{-1}$ year^{-1}.

The maximizing effort is $E_{opt} = 0.5\frac{\left(qK-\frac{c}{p}\right)}{q^2K/r} \approx 278$ skates. The corresponding yield is Y_{opt} = 13.608×10^6 kg year^{-1}.

Problem M13

The dynamics of the fin whale (*Balaenoptera physalus*) is reasonably well described by the following generalized logistic model (N is the whale numbers)

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)^a$$

Problem M13 The fin whale

with $r = 0.06$ year^{-1}, $K = 400,000$ and $a = 0.143$. This stock is now protected, because it has been overexploited. Assume that, once the stock recovers (which will take several decades), hunting is permitted again, but is regulated. Implement a rational management scheme based on the information that the catchability coefficient is $q = 1.3 \times 10^{-5}$ (hunting days)$^{-1}$, the price of one whale is €40,000, the opportunity cost of one hunting day is €4,000.

Find the effort E_0 that maximizes the sustainable profit, and the corresponding profit. Suppose you want to utilize a tax τ (which is levied on each caught whale) as a regulation tool. Calibrate τ so that effort stabilizes to the same E_0 you have already calculated.

Solution
The model describing the dynamics of the whale stock is

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)^a - qEN$$

and therefore, given an effort E, the corresponding stock at equilibrium is

$$N_{eq} = K\left(1 - \left(\frac{q}{r}E\right)^{1/a}\right)$$

The sustainable profit is thus given by

$$P(E) = pqKE\left(1 - \left(\frac{q}{r}E\right)^{1/a}\right) - cE$$

and the optimal effort is found by setting $dP/dE = pqK - \frac{1}{a}pqK\left(\frac{q}{r}\right)^{1/a}E^{1/a} - c = 0$, namely

$$E_{opt} = \frac{[a(pqK - c)]^a}{(pqK)^a(q/r)} = 3{,}485 \text{ hunting days}$$

The corresponding number of whales at equilibrium is N_{opt} = 343,900, the yield is Y_{opt} = 15,581 whales year^{-1}, and the profit is P_{opt} = 609.293 million euros.

The tax τ can be found by setting $(p - \tau)\, Y_{opt} - cE_{opt} = 0$. The resulting tax would be τ = € 39,105 per whale.

Problem M14

Consider an open-access renewable resource, which is simply regulated by imposing a tax on the net profit obtained by each economic operator exploiting the resource. Comment on the efficacy of such a regulation method by using H. S. Gordon's theory on open-access resources.

Solution

The total profit to the exploiters would be

$$P(E) = pY - cE - \tau(pY - cE) = (1 - \tau)(pY - cE).$$

Easy to see that such a tax would be totally ineffective.

Problem M15

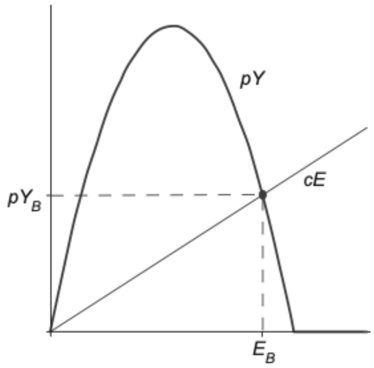
One of the main tenets of social welfare is progressive taxation. Assume that decision makers want to levy a progressive tax on the amount of biomass removed. The tax is structured as follows

$$\tau = \begin{cases} 0 & \text{if } Y \leq Y_0 \\ \tau_{\max}\dfrac{Y - Y_0}{Y} & \text{if } Y > Y_0 \end{cases}$$

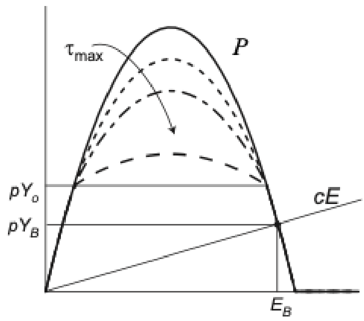
where τ is the amount that is levied on each unit of biomass being harvested and Y_0 is the biomass yield below which the exploiters are exempted from paying taxes. Discuss the efficacy of this regulation method by using a Gordon-Schaefer model (logistic demography, harvest proportional to effort times biomass, and bionomic equilibrium).

Solution

With no tax applied we would have the bionomic equilibrium with $N_B = c/(pq)$ and $E_B = \frac{r}{q}\left(1 - \frac{c}{pqK}\right)$. The yield would be $Y_B = \frac{cr}{pq}\left(1 - \frac{c}{pqK}\right)$ and the situation is the one shown in the graph below



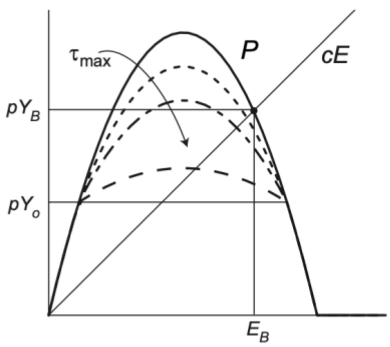
The proposed regulatory taxation depends on two parameters: Y_0 and τ_{max}. First assume that $Y_0 < Y_B$, i.e. $Y_0 < \frac{cr}{pq}\left(1 - \frac{c}{pqK}\right)$. In this case, as shown in the graph below, the tax is completely ineffective because the resulting equilibrium coincides with the unregulated bionomic equilibrium, whatever the value of τ_{max}.



If, instead, $Y_0 > Y_B$, the resulting regulated bionomic equilibrium is given by the equation

$$pY - \tau_{max}(Y - Y_0) - cE = 0$$

and the result is depicted in the graph below. The new bionomic effort is below the unregulated bionomic effort and the higher τ_{max} the more effective the tax.



Problem M16

In the forested areas of North America the beaver (*Castor canadensis*), which is no longer hunted for its fur, is becoming an important factor of nuisance to timber production. In fact, this herbivore can fell mature trees up to a diameter of 40 cm. A cost-benefit analysis estimated the annual damage of one beaver to be \$45. Assume that the local authority wants to implement a culling policy in an area where the beaver carrying capacity is 1 individual km^{-2} and the intrinsic instantaneous rate of demographic increase is 0.3 year^{-1}. Culling is expensive and the cost C can be evaluated as dollars per year per km^2. C can be assumed to be proportional to the inflicted mortality rate m [year^{-1}] and inversely proportional to the beaver density N [No. of beavers km^{-2}], because capturing beavers is harder when their density is lower. Specifically, assume $C = 2.5$ m/N.

Find the best mortality rate, namely the one that minimizes the sum of the damages to timber plus the cost of culling. Estimate the beaver density at equilibrium corresponding to the optimal culling rate.

Problem M16 The beaver

Solution

The dynamics of the beaver population is provided by the equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - mN$$

and the nontrivial equilibrium is $N_{eq} = K(1\text{-}m/r)$. Therefore the damage D at equilibrium is thus

$$D = \alpha K\left(1 - \frac{m}{r}\right) + \frac{\beta}{K}\frac{m}{\left(1 - \frac{m}{r}\right)}$$

where α = \$45 and β = 2.5. We can then find the value of m that minimizes the damage by setting $dD/dm = 0$, that is

$$-\frac{\alpha K}{r} + \frac{\beta}{K}\frac{1}{\left(1 - \frac{m}{r}\right)^2} = 0$$

From which $m_{opt} = r\left(1 - \sqrt{\frac{\beta r}{\alpha} K}\right) = 0.26$ year^{-1}.

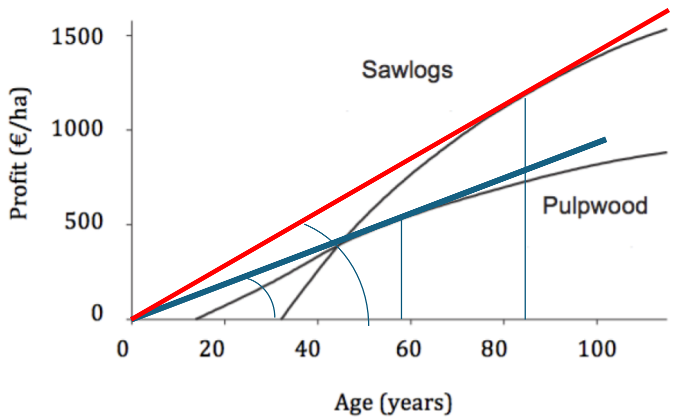
Problem M17

A tree species can be harvested for producing either pulpwood or sawlogs. The figure reports the net profit that can be obtained from clear-cutting a forest lot at different ages. Assume that the discount rate is zero and find out the optimal rotation periods for the two alternatives. Evaluate whether it is more convenient to produce pulpwood or sawlogs.

Problem M17 Net profit per hectare as a function of age for a tree species that can be harvested for producing pulpwood or sawlogs

Solution

The optimal rotation period is where the marginal profit equals the average profit. From the graphical viewpoint it corresponds to finding the tangent to the profit curve that goes through the origin (see the graph below).



Therefore, the optimal rotation period for pulpwood is T_p = 58 years, for sawlogs T_s = 85 years. However, $P(T_p)/ T_p < P(T_s)/ T_s$ and therefore it is more convenient to produce sawlogs.

Problem M18

The loblolly pine (*Pinus taeda*) is a conifer that is harvested in North America for its sawwood. The biomass of merchantable timber is a function of loblolly age and can be assumed to be given by

$$B(T) = \begin{cases} 0 & \text{if} \quad T < 6 \\ 9.18T - 0.077T^2 - 55.09 & \text{if} \quad T \geq 6 \end{cases}$$

where T is age in years and B is the biomass (tonnes/acre) that can be harvested and sold. The selling price of one merchantable tonne of pine is about \$30. The yearly maintenance cost of one acre of forest is \$9, while the cost of clearcutting and replanting one acre is \$125 independently of the age of the trees.

Assume that the discount rate is zero and find the optimal rotation period of the loblolly pine plantation.

Problem M18 The loblolly pine

Solution
The profit $P(T)$ obtainable by harvesting at the age T is

$$P(T) = pB(T) - c_A - c_M T = p(\alpha T - \beta T^2 - \gamma) - c_A - c_M T$$

from which we derive that the optimal rotation period is

$$T_{opt} = \sqrt{\frac{p\gamma + c_A}{p\beta}} = 27.74 \cong 28 \text{ years.}$$

Problem M19

You want to manage farmed salmon in Norway. In particular you would like to know the optimal period at which you should harvest all the salmon that you initially stocked as baby salmon in a farming cage. The data are as follows:

- In each cage you stock 100 baby salmon and the cost of purchasing and stocking each baby is €0.8;
- The mortality is practically zero, so you can neglect it;
- The salmon grow approximately in a quadratic way and the weight w of each salmon (grams) as a function of age T (days) is $w = 15T/(1 + 0.001T)$;
- The daily cost of feeding and managing the cage is 0.011 €day^{-1};
- The selling price of an adult salmon is 6 €kg^{-1}.

Find the optimal age at which you should harvest the salmon and stock another 100 babies, assuming that the rate of discount is zero. Calculate the profit from the cage.

Solution
The profit $P(T)$ obtainable by harvesting at the age T is
$P(T) = S_0\,(pw(T) - c_A - c_M T) = S_0\,(p\alpha T/(1+\beta T) - c_A\,) - c_M T$
from which we derive the optimal

$$T_{opt} = \frac{\beta c_A + \sqrt{\beta c_A p\alpha}}{\beta(p\alpha - \beta c_A)} = 104 \text{ days.}$$

The corresponding average profit is 7.37 € day^{-1} and the total profit from each cage at the end of the rotation period is $P(T_{opt})$ = €767.38

Problem M20

Many seal populations are protected by law and international treaties. Fishermen, however, have often complained that the increase of seal numbers damages their catches, because these marine mammals subtract fish biomass that, without increased protection, would actually end up in their nets. Investigate the problem with reference to a constant-recruitment fish population in which the weight w of each individual fish varies with age τ [years] according to the following law

$$w(\tau) = w_{\max}\,(1 - \exp(-k\tau))$$

where

$$w_{\max} = 10 \text{ kg}$$
$$k = 0.1 \text{ year}^{-1}$$

while survival from age 0 (recruitment age) up to age τ follows the following law

$$p(x) = \exp\,(-\mu\tau)$$

with μ being a constant. Assume that cost of fishing effort is negligible and that the mortality rate without seals is $\mu_0 = 0.1\,\text{year}^{-1}$, while the mortality rate with seals is $\mu_S = 0.2\,\text{year}^{-1}$. Calculate the optimal age to be selected by the fishing gear in the two cases by assuming that the fishermen optimize the sustained biomass yield. Calculate the variation of yield between the two cases.

Solution

The biomass $B_i(\tau)$ of the fish increases with the fish age τ depending on the presence of seals (i = 0, S) according to the equation

$$B_i(\tau) = Rw_i(\tau)p(\tau) = Rw_{max}(1 - \exp\,(-k\tau))\exp(-\mu_i\tau).$$

where R is the yearly recruitment. Since the harvesting cost is negligible and the recruitment is continuous, the corresponding sustained yield is obtained by maximizing the biomass and applying a very large (theoretically infinite) effort corresponding to the optimal age.

The age $\tau_{max,i}$ where $B_i(\tau)$ is largest is derived by setting $dB_i(\tau)/d\tau = 0$. Thus, we obtain $\tau_{max,i} = \frac{1}{k}\ln\left(1 + \frac{k}{\mu_i}\right)$, that is $\tau_{max,0}$ = 6.93 years and $\tau_{max,S}$ = 4.05 years.

Correspondingly, the harvested biomass is

$$B_i(\tau_{max,i}) = Rw_{max}(1 - \exp\,(-k\tau_{max,i}))\exp(-\mu_i\tau_{max,i}).$$

The percent variation of yield is thus given by

$$1 - \frac{Rw_{max}(1-\exp\,(-k\tau_{max,S}))\exp(-\mu_i\tau_{max,S})}{Rw_{max}(1-\exp\,(-k\tau_{max,0}))\exp(-\mu_i\tau_{max,0})} = 0.41.$$

In conclusion, the complaints of fishermen are not unjustified because this fish-stock yield with seals decreases by 41%.

Problem M21

You have been requested to manage aquaculture in the lagoon of Stillwater. Every year, juveniles of the fantasy species *Argenteus bonissimus*, which is very much appreciated by gourmets, are recruited from the open sea to the lagoon. The weight w [kg] of the fish increases with age τ [years] according to

$$w(\tau) = w_{\max}\,(1 - \exp(-k\tau))$$

where $w_{\max} = 2\,\text{kg}$ and k is a growth coefficient which depends on the food provided to *A. bonissimus*. It equals $0.25\,\text{year}^{-1}$ if a highly caloric fish-meal C_1 is fed to the fish or $0.15\,\text{year}^{-1}$ if a poorer food C_2 is employed. The mortality rate of the fish is constant with age and equal to $0.1\,\text{year}^{-1}$. Also, there is no reproduction in the lagoon and the recruitment is constant and equal to 100,000 juveniles per year. The following economic data are available:

(i) the selling price of 1 kg of *A. bonissimus* is €10;
(ii) the harvesting cost is negligible whatever the harvested biomass and the mesh of the gear;
(iii) the annual cost of using C_1 is €400,000 while that of C_2 is €150,000.

Determine the best harvesting policy (selected age) and the optimal fish-meal.

Solution
Let $\tau_{max,i}$ be the optimal age for policy i (i = 1, 2).
Following a procedure similar to that of the previous exercise, we obtain $\tau_{max,i} = \frac{1}{k_i}\ln\left(1 + \frac{k_i}{\mu}\right)$, that is $\tau_{max,1}$ = 5.01 years and $\tau_{max,2}$ = 6.11 years. Correspondingly, the profits are P_1 = €465,515 year^{-1} and P_2 = €501,460 year^{-1}.
In conclusion, it is more convenient to utilize the poorer food and use a fishing gear that selects fish 6 years-old and older.

Problem M22

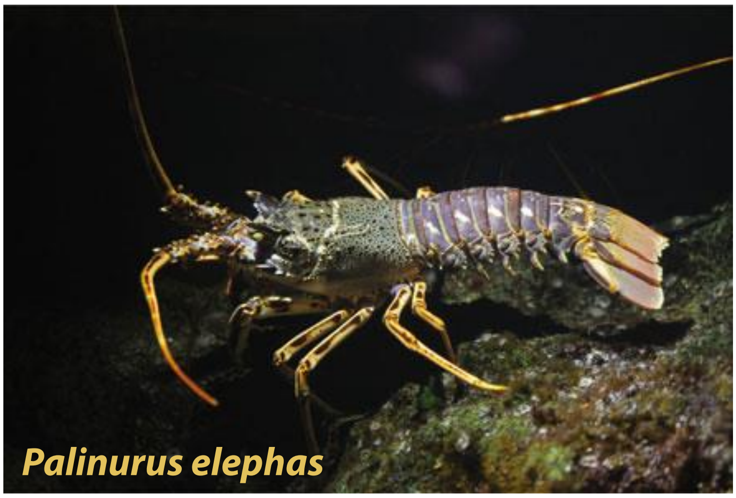
The spiny lobster (*Palinurus elephas*) is widespread in western Mediterranean and is an important resource for Sardinian fisheries. Bevacqua et al. (2010) and Tidu et al. (2004) have determined the following relationship linking the age x (years) to the fresh weight w (g) of lobsters in Sardinia:

$$w(x) = w_{\max}(1 - \exp(-kx))^3$$

with $w_{max} = 1300$ g and $k = 0.16$ year^{-1}.

Also, Bevacqua et al. (2010) estimated that the mortality rate μ of lobsters is constant and equal to 0.27 year^{-1}. You want to manage a stock of lobsters for which the recruitment to age 0 is constant and equal to 10,000 young lobsters. Suppose that

Problem M22 The spiny lobster



the cost of fishing effort is negligible whatever the harvested biomass and the mesh of the fishing gear. The selling price of 1 kg of lobsters is €60. Determine

(a) the best age to be selected by the fishing year,
(b) the corresponding harvested biomass, and the corresponding economic return.

Solution
The solution is very similar to those of the previous exercises. The optimal age is $x_{opt} = \frac{1}{k}\ln\left(1 + \frac{3k}{\mu}\right) = 6.39$ years. The corresponding harvested biomass is 607.78 kg and the profit is €36,467.