1. The density (No. per square metre) of the clam *Venetapes polytechnica* has been measured for a few years in the Nowhere Sea. Data are reported here below

Year	1985	1986	1987	1988	1989	1990	1991
Abundance	21	25	20	23	27	24	29

Based on this information:

1. Estimate the median finite growth rate of the population;
2. Estimate the variance of the environmental log-noise;
3. Assume that the environmental noise is lognormal and use the 1991 abundance as the initial condition to calculate future probability densities of the population abundance; what is the probability density of the logarithm of abundance in 1998?

Solution:

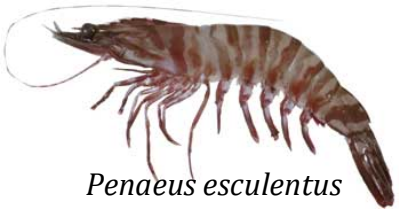
2. The red muskrat is an alien species for the Austramerican continent. The environmental conservation agency wants to at least slow down its continuous expansion. To this end, the muskrat movement ability is first studied. Ten rats are radio-collared and released in the same location on April 15, 2016. By suitably fencing the territory surrounding the location point, the rats are allowed to move only in the direction North-South. On May 5, 2016, all the rats are recaptured at various distances. The data are reported in the table below (positive distances for movements towards North, negative distances for movements towards South).

Individual	Distance (m)
1	-320
2	570
3	-1400
4	-600
5	300
6	-1100
7	-760
8	800
9	1500
10	1000

1. Use the data to estimate the diffusion coefficient of the muskrat.
2. The asymptotic speed of expansion of the muskrat in Austramerica has reached about 4 km year^{-1}. Also, its demographic increase can be assumed to be exponential. Estimate the instantaneous and the finite rate of increase of the red muskrat.
3. The environmental agency wants to slow down the rat expansion by means of culling. The goal is to reduce the expansion speed to one fourth of the current speed. Calculate how large the mortality rate m (year^{-1}) due to culling should be in order to achieve the agency's goal.

Solution:

3. The northern prawn fishery is one of the most important Australian fisheries. The tiger prawns *Penaeus esculentus* and *P. semisulcatus* are two most prominent species subject to harvesting. You-Gan Wang and Davide Die (Marine and Freshwater Research, 1996) determined the stock-recruitment curves for the two species from available data.



Penaeus esculentus

If B_k is the total biomass (tonnes) in year k, we can approximate the curves by a Beverton-Holt model as follows:

$$B_{k+1} = \lambda B_k/(1+\alpha B_k)$$

Penaeus esculentus: $\lambda = 1.6$ $\alpha = 0.00019$

P. semisulcatus: $\lambda = 2.16$ $\alpha = 0.0004$.

The effort is measured as boat-days (that is the number of operating boats times the days they are operating in a given year). From the estimates of the two researchers we can derive the catchability coefficients for the two prawns:

Penaeus esculentus: $q = 3\times10^{-4}$ (boat-days)$^{-1}$

Penaeus semisulcatus: $q = 3.5\times10^{-4}$ (boat-days)$^{-1}$

For each of the two prawns, determine (1) the Maximum Sustainable Yield, (2) the effort that provides the MSY and (3) the corresponding prawn biomass.

Solution:

4. Zika virus is an emerging mosquito-borne virus that infects and causes disease in humans. Although symptoms are usually mild, there is evidence that the infection of women during a critical part of pregnancy can lead to the development of microcephaly in the unborn child. The most important disease vector is the Yellow Fever mosquito, *Aedes aegypti*.

C. Caminade et al. (PNAS, 2017) have studied the dynamics of the disease in relation to the climate of the countries hit by an epidemic of the virus. They estimated the following parameters for a country with an average temperature of 25 °C.

- a = number of bites per mosquito per unit time = 0.2 day^{-1}
- m = number of female mosquitoes per human host = 50
- b = probability of transmission of infection from infectious mosquitoes to humans per bite = 0.5
- ξ = mortality rates of mosquitoes = 0.19 day^{-1}
- recovery time from the disease = 7 days
- c = probability of transmission of infection from infectious humans to mosquitoes per bite = 0.1

1. From the above parameters derive β, the mosquito-to-human transmission rate and ψ, the human-to-mosquito transmission rate.
2. Write down a Ross model for the zika virus describing the dynamics of the prevalence of infected humans (U) and that of infected mosquitoes (M).
3. Calculate the basic reproduction number and establish whether the disease can establish in the country. If it can, calculate the prevalence of both humans and mosquitoes at equilibrium.

Solution: