1. The natural habitat of the butterfly *Euphydria felina* consists of tropical forest fragments. Local entomologists have estimated that the fraction of occupied fragments is 70%. Subsequently a certain number of fragments is fenced so that no immigration or emigration can occur. In this way it is possible to calculate that the average extinction time of a local subpopulation is 4 years.

(a) Using these data, calculate the coefficient *c* of fragment colonization and write down the Levins equation describing the dynamics of the metapopulation.

(b) Then, establish the fate of the butterfly if 25% of the habitat were permanently destroyed.

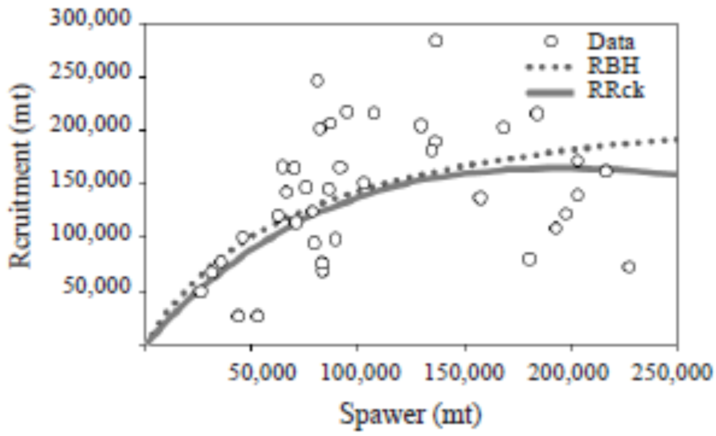
(c) Also, establish the fate of the butterfly when, because of climate change, the metapopulation is subject to environmental catastrophes that wipe out a local subpopulation every 3 years in the average.

Solution:

2. The spotted goose (*Anser maculatus*) is a bird threatened by extinction. Its average lifetime is about 8 years. The average number of newborns produced per pair is about 6 and the sex ratio is 1:1. Only 15% of the little birds will survive to adulthood. Using these data, calculate the mortality rate μ and the natality rate ν in years^{-1}. Environmentalists want to reintroduce the goose into the Golden Plane reserve by releasing 12 adult geese (6 males and 6 females). Estimate the long-term probability of persistence for the new population at Golden Plane.

Solution:

3. Zhang et al. (Journ. Korean Soc. Fish Tech., 2012) describe the dynamics of the Korean stock of Japanese horse mackerel (*Trachurus japonicus*). From data they have obtained a stock-recruitment relationship linking the biomass (in metric tons) of one generation (B_t) to the biomass of the subsequent generation (B_{t+1}).



This relationship can be approximated as follows

$B_{t+1} = A\,(1 - \exp(-\lambda B_t))$ with $A = 1.9\times10^5$ tons and $\lambda = 1.42\times10^{-5}$ ton^{-1}.

(a) Find the constant escapement policy that maximizes the sustainable yield of the mackerel stock.
(b) Calculate the corresponding MSY.
(c) Calculate the fraction u of the recruitment that is taken by means of the optimal policy at equilibrium.
(d) Assume that effort is measured in number of operating vessels, that the catchability coefficient is $q = 0.02$ (No. of vessels)$^{-1}$ year^{-1} and that there are 150 fishing vessels that operate during a fishing season of duration T (months); how long should T be for actually harvesting the fraction u ?

Solution:

4. Cholera is a microparasitic disease transmitted through exposure to water contaminated by the bacterium *Vibrio cholerae*. The dynamics of bacterial concentration can be described by the simple model:

$$\dot{B} = pI - \mu_B B$$

where p=10^{-6} (No.infected)$^{-1}$ day^{-1} is the contamination rate, μ_B = 0.2 day^{-1} is the death rate of *V. cholera*, I is the number of infected and B is the normalized concentration of bacteria in the water (namely the concentration of bacteria divided by their carrying capacity). Since the dynamics of *V. cholerae* is faster than human dynamics, bacterial concentration can be assumed to be at equilibrium. In other words, we can assume that bacterial concentration is instantaneously linked to the number of infected via the relationship

$$B(t) = pI(t)/\mu_B \ .$$

Use this relationship to analyse the dynamics of cholera in a human community in which the recruitment of new susceptible individuals (due to either newborns or migration from nearby communities) is a constant flow w. Use an SI model without immunity characterized by:

- average life time of susceptibles: 70 years
- constant flow w = 5 individuals day^{-1}
- infection rate = $\beta B/(1+B)$ with β = 1 day^{-1}
- recovery rate γ = 0.2 day^{-1} .

Based on these data, (a) write down the SI model for the cholera dynamics. Then, consider a population that at the endemic equilibrium is characterized by 10,000 infected individuals. For this population (b) compute the equilibrium prevalence of the disease and (c) determine the disease-related death rate α.

Solution: