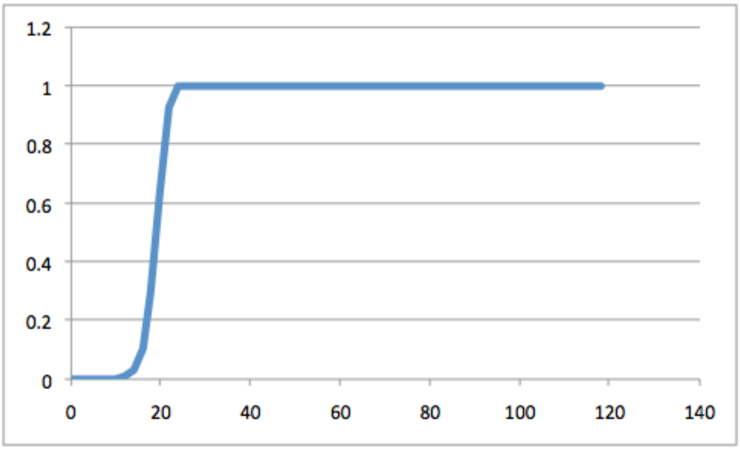
1. A small population of red geese has been studied for several years. From a demographic viewpoint the population is more or less at equilibrium with a size of about 50 individuals. However, the effective population size N_e is not known. It can be inferred, though, from the loss of heterozygosity H, which has been measured in some selected years (table below).

Year	H
1995	0.7
2000	0.66
2004	0.61
2009	0.59
2012	0.55
2016	0.54

The generation time of the geese is 2 years. Estimate the effective population size.

Solution:

2. The habitat of the wild sheep *Ovis polytechnicus* has been decreasing steadily in the past two centuries. Therefore, a few decades ago several square reserves of different size have been established in order to protect many wild sheep populations. Some of these populations have become extinct while others have fared well and have reached their carrying capacity. Conservation ecologists have observed that the probability of reaching the carrying capacity depends basically on the reserve area (km^2) according to the following relationship.



The environmental agency decides to reintroduce *O. polytechnicus* in the big Vinci Park (160 km^2) where the sheep is not present. The agency officers want to estimate the expansion speed of the sheep. To that end, they release 20 collared sheep on April 1, 2015. After 1 month they find out that all the sheep have survived and 15 are inside a circle with radius 3 km. Estimate the diffusion coefficient *D* from this information.
Then, assume that the intrinsic growth rate within Vinci Park is the same as the one recorded in the smaller reserves and estimate the expansion speed of *O. polytechnicus*.

Solution:

3. The silvergrass *Miscanthus giganteus* is an energy crop that is widely used for its rapid growth. The biomass yield (Mg ha^{-1}) in UK according to G. M. Richter et al. (*Biomass and Bioenergy*, 2016) can be assumed to be given approximately by

$$B(T) = \begin{cases} 0 & \text{if } T < 0.5 \\ \dfrac{10(T-0.5)}{1.5+T} & \text{if } T \geq 0.5 \end{cases}$$

where T is the age in years and B is the biomass that can be harvested and sold. The selling price of one ton of the crop is about €30. The cost of clearcutting and replanting one hectare is €200 independently of the age of the grass.
Assume that the discount rate is zero and find the optimal rotation period of the silvergrass plantation.

Solution:

4. During summer 2007 Italy has experienced the first large outbreak caused by Chikungunya virus (CHIKV) documented in a temperate climate country. CHIKV is an arthropod–borne virus which can be transmitted to humans by *Aedes* mosquitoes, in particular by the tiger mosquito *Aedes albopictus* which was introduced into Italy in the early 1990's.
P. Poletti et al. (*PLoS One*, 2011) have developed a stage-structured model for the dynamics of the disease outbreak in Emilia-Romagna, which can be simplified to a standard Ross model. They estimated the following parameters for a temperature of 25 °C.

- a = number of bites per mosquito per unit time = 0.09 day^{-1}
- b = probability of transmission of infection from infectious mosquitoes to humans per bite = 65%
- average life time of adult mosquitoes = 32 days
- recovery time from the disease = 4.5 days
- c = probability of transmission of infection from infectious humans to mosquitoes per bite = 80%
- R_0 = basic reproduction number = 3.3

Write down a Ross model for the CHIKV epidemic describing the dynamics of the prevalence of infected humans (U) and that of infected mosquitoes (M).
From the above parameters derive m = number of female mosquitoes per human host. Then calculate the prevalence of both humans and mosquitoes at equilibrium.

Solution: